

# Natural $\mu$ -term generation in supergravity scenario

Ryuichiro Kitano<sup>ab,\*</sup> and Nobuchika Okada<sup>a,†</sup>

<sup>a</sup>*Theory Group, KEK, Oho 1-1, Tsukuba, Ibaraki 305-0801, Japan*

<sup>b</sup>*Department of Particle and Nuclear Physics, The Graduate University for Advanced Studies,  
Oho 1-1, Tsukuba, Ibaraki 305-0801, Japan*

## Abstract

We discuss a natural way to generate the  $\mu$ -term in supergravity scenario. Once the supergravity effects are taken into account, the vacuum expectation values (VEVs) of the heavy fields are in general shifted from the values in the supersymmetric limit. We note that this fact is independent of any Kahler ansatz and the values of the VEV shifts are of the order of the gravitino mass. As an example, an explicit model is presented, in which both of the  $\mu$ -term and the  $B$ -term of the electroweak scale are generated by the VEV shifts through the supergravity effects. This model is a kind of the next to minimal supersymmetric standard model, but there is no light standard model singlet field. Also, we emphasize that our discussion can be naturally applied to the supersymmetric grand unified theory.

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\*email: ryuichiro.kitano@kek.jp

†email: okadan@camry.kek.jp

# 1 Introduction

Supersymmetry (SUSY) is the most attractive candidate for the theory above TeV scale. It ensures stability of the weak scale  $m_W$  [1], and predicts the gauge coupling unification [2] which naturally indicates a beautiful unification of the standard model gauge interactions to the SU(5) grand unified theory (GUT) [3]. The radiative breaking of the electroweak symmetry is also an interesting feature [4, 5].

We know that SUSY is broken because none of superpartners has not been observed yet. Therefore we should include the SUSY breaking mechanism in the theory. Investigating the SUSY breaking scenario is an important topic in particle physics [6]. One of the successful SUSY breaking scenarios is the supergravity scenario [7, 8], in which SUSY is broken in the hidden sector and the SUSY breaking information communicate the visible sector through the supergravity interactions. In this scenario, if we assume the SUSY breaking scale of order  $10^{11}$  GeV, soft breaking terms in the visible sector are obtained to be of the order of weak scale so that the electroweak scale is stabilized.

However, we have a less understanding feature in SUSY models, that is so called the  $\mu$ -problem [9]. In the minimal supersymmetric standard model (MSSM), the superpotential have a bilinear term in the Higgs sector such as  $W \supset \mu H_1 H_2$ , where  $H_1$  and  $H_2$  are Higgs doublets, and  $\mu$  is a parameter of dimension one. From the naturalness point of view, this supersymmetric parameter  $\mu$  would be of the order of the GUT ( $M_{\text{GUT}}$ ) or Planck ( $M_{\text{Pl}}$ ) scales or zero by some symmetry reason. However, from a claim for the correct electroweak symmetry breakdown, this  $\mu$  parameter must be the same order as the weak scale, so that  $\mu \sim M_{\text{GUT}}$  or  $M_{\text{Pl}}$  is not allowed. A vanishing  $\mu$  parameter is also forbidden because it leads to a massless chargino being excluded by experiments [10]. The mysterious question is why the “supersymmetric” parameter  $\mu$  is the same order as the weak scale whose origin is “SUSY breaking” [9].

There have been many attempts towards this problem. Giudice and Masiero considered a possibility of existence of higher dimensional interaction terms between Higgs fields and hidden sector fields in Kahler potential, and these terms induce the  $\mu$ -term through SUSY breaking [11]. Other models have been also considered e.g. so called next to minimal supersymmetric standard model (NMSSM) [12], connecting Higgs fields with hidden sector in the superpotential [13], and imposing additional symmetries [14].

In this paper, we consider another way to produce the  $\mu$ -term in the supergravity scenario in which neither particular Kahler potential nor particular hidden sector is required. We consider VEV shifts of heavy fields through the supergravity interactions. In the supersymmetric limit, the VEVs for the heavy fields are determined by the vanishing F-term and D-term conditions. However, when we switch on the supergravity interactions, the potential is deformed and VEVs for the heavy fields are shifted from that in the supersymmetric limit. The values of the VEV shifts are found to be of order gravitino mass  $m_g (\sim m_W)$  [8]. We propose a scenario that the  $\mu$ -term is exactly vanishing in the supersymmetric limit, but generated by the VEV shift of heavy fields.

This paper is organized as follows: In Section 2, we review the VEV shifts in supergravity theories. In Section 3, we present an explicit model of  $\mu$ -term generation by using VEV shifts of heavy fields and discuss the application to GUT theory. In Section 4, we give our conclusions.

## 2 VEV shifts in supergravity

In this section, we review VEV shifts of heavy fields in the supergravity scenario. The values of the VEV shifts are found to be of the order of  $m_g$  [8].

The scalar potential of supergravity is given by [15]

$$V = e^K \left[ \left( \frac{\partial W}{\partial z_i} + \frac{\partial K}{\partial z_i} W \right) g^{i\bar{j}} \left( \frac{\partial W^*}{\partial z_j^*} + \frac{\partial K}{\partial z_j^*} W^* \right) - 3|W|^2 \right] + (\text{D-terms}) , \quad (1)$$

where we take a unit  $8\pi G_N = 1$ ,  $K$  and  $W$  are Kahler potential and superpotential, respectively,  $z_i$  represents scalar components of chiral superfields, and  $g^{i\bar{j}}$  is Kahler metric:

$$g_{i\bar{j}} = \frac{\partial^2 K}{\partial z_i \partial z_j^*} . \quad (2)$$

In supergravity scenario, the superpotential is divided into visible and hidden sectors as follows:

$$W = W_{\text{vis}} + W_{\text{hid}} . \quad (3)$$

When we claim the SUSY is broken in the hidden sector and the vanishing cosmological constant conditions,  $\langle V \rangle = 0$  and  $\langle W_{\text{vis}} \rangle = 0$ , the gravitino mass is given by  $m_g = \langle e^{K/2} W_{\text{hid}} \rangle \sim \langle W_{\text{hid}} \rangle \sim m_W$ .

In the visible sector, the field VEVs in the supersymmetric limit are determined by the vanishing F-term condition such as  $\partial W_{\text{vis}}/\partial z_i|_{z_i=z_i^0} = 0$ . However, note that in the potential given in eq.(1), the VEVs are shifted by the SUSY breaking effect. Here, we parameterize the VEV shift of the visible sector fields as

$$z_i = z_i^0 + \delta z_i . \quad (4)$$

Assuming  $\delta z_i \sim O(m_g) \sim \langle W_{\text{hid}} \rangle$ , and expanding the potential with respect to  $\langle W_{\text{hid}} \rangle$  and  $\delta z_i$ , the leading order terms are given by

$$V \simeq e^K \left( \frac{\partial^2 W_{\text{vis}}}{\partial z_i \partial z_k} \Big|_{z^0} \delta z_k + \frac{\partial K}{\partial z_i} \Big|_{z^0} \langle W_{\text{hid}} \rangle \right) g^{i\bar{j}} \left( \frac{\partial W_{\text{vis}}^*}{\partial z_j^* \partial z_l^*} \Big|_{z^0} \delta z_l^* + \frac{\partial K}{\partial z_j^*} \Big|_{z^0} \langle W_{\text{hid}}^* \rangle \right) . \quad (5)$$

The stationarity condition  $\partial V/\partial(\delta z_i) = 0$  leads to

$$\delta z_i = - \left( \frac{\partial^2 W_{\text{vis}}}{\partial z_i \partial z_j} \Big|_{z^0} \right)^{-1} \frac{\partial K}{\partial z_j} \Big|_{z^0} \langle W_{\text{hid}} \rangle . \quad (6)$$

This is generally of the order of  $m_g$ . The inversibility of  $\partial^2 W_{\text{vis}}/\partial z_i \partial z_j|_{z^0}$  means that the fields  $z_i$ 's are all massive. We can see that VEV shifts of heavy fields are of the order of  $m_g$ .

### 3 Models

In this section, we propose an explicit model in which the  $\mu$ -term is generated by the VEV shifts discussed in the previous section. Although the  $\mu$ -term is absent in the supersymmetric limit, non-zero  $\mu$ -term emerges through the VEV shifts of heavy fields.

The visible sector superpotential is given by

$$W = W_{\text{MSSM}} + \lambda_H S H_1 H_2 + \lambda_N S (N^2 - m^2) , \quad (7)$$

where  $W_{\text{MSSM}}$  is the superpotential in MSSM except for the  $\mu$ -term,  $S$  and  $N$  are standard model singlet chiral superfields,  $m$  is a mass parameter of order GUT or Planck scale, and  $\lambda_H$  and  $\lambda_N$  are dimensionless coupling constants. This superpotential is general under the standard model gauge group and R-symmetry where we assign R-charge 2 for  $S$ , 0 for  $H_1$ ,  $H_2$ , and  $N$ , and 1 for all other chiral superfields in MSSM. Here we omit bilinear

term  $SN$  by the field redefinition of  $N$ .<sup>1</sup> In the supersymmetric limit, the field VEVs are given by

$$\langle S \rangle = 0, \quad \langle N \rangle = m, \quad (8)$$

where we assume  $SU(2)_L \times U(1)_Y$  unbroken vacuum i.e.  $\langle H_1 \rangle = \langle H_2 \rangle = 0$ . In this stage, the Higgs doublets are massless and the field  $S$  and  $N$  are both superheavy.

Let us estimate the VEV shifts for  $S$  and  $N$  by substituting eq.(7) into eq.(6). The second derivatives of the superpotential (which is mass matrix for  $S$  and  $N$ ) are given by

$$\frac{\partial^2 W_{\text{vis}}}{\partial z_i \partial z_j} = \begin{pmatrix} 0 & 2\lambda_N m \\ 2\lambda_N m & 0 \end{pmatrix}, \quad (9)$$

where  $z_1 = S$  and  $z_2 = N$ . For simplicity, we assume that the zero-th order of the Kahler potential in the power series of  $z_i/M_{\text{Pl}}$ , is of the canonical form,  $K = S^*S + N^*N$ . The higher order terms can be neglected if  $z_i/M_{\text{Pl}} \ll 1$ .<sup>2</sup> Now we can obtain the leading order VEV shifts from eqs.(6), (8), and (9) as follows:

$$\delta S = -\frac{1}{2\lambda_N} \langle W_{\text{hid}} \rangle, \quad \delta N = 0. \quad (10)$$

Note that  $S$  acquires the VEV of order  $m_g$ , which means that the  $\mu$ -term of order  $m_W$  is successfully generated. More explicitly, we can write down the low energy effective potential by integrating out the  $S$  and  $N$  fields as

$$V_{\text{eff}} = \lambda_H^2 |\delta S|^2 (|H_1|^2 + |H_2|^2) - 2\lambda_H \lambda_N (|\delta S|^2 + |\delta N|^2) H_1 H_2 + \text{h.c.} \\ + (\text{D-terms}) + (\text{soft terms}), \quad (11)$$

where the last term denotes the soft terms dependent on the Kahler potential. In the above equation, we can see that the  $B$ -term ( $\mathcal{L} \supset -B\mu H_1 H_2$ ) can be also generated,  $B\mu \sim O(m_g^2)$ , which is suitable for electroweak symmetry breaking.

Note that the low energy effective theory of our model after integrating out heavy singlet fields  $S$  and  $N$  is just MSSM, and there is no light fields except for the MSSM particle contents. This is the crucial difference from usual NMSSM in which there is a light singlet field in low energy superpotential [12]. In NMSSM, a standard model singlet field

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<sup>1</sup> Although this field redefinition may induce a linear term in the Kahler potential, it does not change our result of  $\mu$ -term and  $B$ -term generation.

<sup>2</sup> This assumption is introduced just for simplicity. Even if  $z_i/M_{\text{Pl}}$  is not small, we can obtain suitable  $\mu$ -term and  $B$ -term proportional to  $m_g$  with the coefficients which depend on Kahler potential.

$X$  is introduced and couples to Higgs fields as  $W_{\text{NMSSM}} \supset XH_1H_2 + X^3$ . The minimization condition for the potential including the soft terms leads to  $\langle X \rangle \sim m_W$  which generates effectively the  $\mu$ -term. However, in supergravity scenario with such a light singlet field, there is a serious problem that the weak scale is destabilized by large tadpole operator induced by supergravity interactions [16] or GUT interactions [5]. The present model does not suffer from this problem because the singlet fields  $S$  and  $N$  are heavy enough to ignore such tadpole terms.

In GUT models, the  $\mu$ -problem is a part of doublet-triplet Higgs mass splitting problem. The two Higgs doublets are embedded into  $\mathbf{5}$  and  $\bar{\mathbf{5}}$  of SU(5). The success of gauge coupling unification requires the SU(2) doublet parts of  $\mathbf{5}$  and  $\bar{\mathbf{5}}$  are light ( $\lesssim m_W$ ) while SU(3) triplet parts are heavy ( $\sim M_{\text{GUT}}$ ). In the minimal SU(5) GUT model, this splitting is accomplished by fine-tuning of the parameters in the GUT breaking sector. The superpotential of the GUT breaking sector is given by

$$W = mH\bar{H} + H\Sigma\bar{H} + V(\Sigma) , \quad (12)$$

where  $H$  and  $\bar{H}$  are  $\mathbf{5}$  and  $\bar{\mathbf{5}}$  Higgs fields, and  $\Sigma$  is an adjoint representation field. The light doublets needs fine-tuning between the Higgs mass parameter  $m$  and  $\langle \Sigma \rangle$  with accuracy of  $10^{-14}$  level. However, if we can find a mechanism in which SUSY vacuum condition requires vanishing doublet Higgs masses, namely, a particular relation between  $m$  and  $\langle \Sigma \rangle$ , the  $\mu$ -term and  $B$ -term can be generated in the correct order by the mechanism discussed in the previous section [5]. As an example of this approach, the model recently proposed by the present authors [17] is very remarkable, in which the doublet-triplet Higgs mass splitting is realized by means of the SUSY gauge dynamics in the supersymmetric limit. We can find that both of the  $\mu$ -term and  $B$ -term of order  $m_W$  are really generated in this model through the VEV shifts by the supergravity effects.<sup>3</sup>

## 4 Conclusions

In conclusion, we presented explicit models in which  $\mu$ -term is generated by the VEV shifts of heavy fields in the supergravity scenario. In the supersymmetric limit,  $\mu$ -term is

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<sup>3</sup> In ref.[17], we mentioned that we cannot obtain  $\mu$ -term of order  $m_W$  according to dimensional analysis. However, in supergravity scenario, the naive analysis is not applicable. We can find that the  $\mu$ -term of order  $m_W$  is really generated in this model. This fact makes the model more impressive.

forbidden by the R-symmetry [18] which is broken in the hidden sector. The VEV shifts of heavy fields are generally of order  $m_g$  without any assumptions for a particular form of Kahler potential or a particular hidden sector. This mechanism requires the presence of a heavy standard model singlet field which couples to Higgs doublets. In GUT models, it can be identified as the GUT breaking field e.g. the standard model singlet component of the  $SU(5)$  adjoint representation field.

In GUT models, the  $\mu$ -problem is connected to the doublet-triplet Higgs mass splitting problem. This problem is one of the most challenging problems in particle physics. The standard model requires SUSY from the naturalness point of view and GUT in order to account for the electric charge quantization of fermions. Although SUSY and GUT are independently required, SUSY surprisingly predicts the gauge coupling unification with the GUT normalization of the  $U(1)_Y$  charge. Therefore the SUSY GUT may be a consistent and attractive theory as the high energy physics. The remained problem is to naturally realize the doublet-triplet Higgs mass splitting. The interesting point of this problem is that the resolution of this problem needs its failure, namely, the  $\mu$ -term for the doublet Higgs fields must not completely vanish, but be of order  $m_W$ . This fact may give us a hint to solve this problem. A natural approach for the doublet-triplet Higgs mass splitting is that the splitting completely successes in the supersymmetric limit, but SUSY breaking effects disturb this success. Our conclusion is that such a scenario is possible in the supergravity scenario.

## Acknowledgments

We would like to thank Yukihiro Mimura for useful discussions. This work was supported by the JSPS Research Fellowships for Young Scientists (R.K.).

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